Strategy of Politics

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What you will get out of this course:

• The basics of game theory. By the end, you will be able to write down and solve simple games.

• An introduction to the formal study of political theory.
What is game theory?

• The “theory of interdependent decision” (Thomas Schelling, an early game theorist and international relations theorist)

• Interdependent decision means that when one actor is making a decision, she has to take into account the likely reactions of others. They do this as well, and so on.

• Can you think of some examples in politics?
Uses of game theory

- Useful for breaking out of potentially unmanageable complexity of situations like the ones just described.
- Many applications developed in the study of domestic political institutions; will also look at areas of international politics.
- Applications to other areas of decision: economics, personal relations, management.
What math skills are needed and learned?

- Need algebra
- Will learn probability theory
- Set theory
- Functions
- Will not use calculus
Flat tire game

Story: A group of students were on a weekend camping trip. They were late returning, and missed a midterm. They told the professor that they could not get back on time because they had a flat tire. She said she would give them an extension if they all gave the same answer (without consultation) to one question: “Which tire?”
Flat tire game

- On a piece of paper, write down one of the following four choices: Passenger front, Passenger rear, Driver front, or Driver rear
- No talking!
- TA will tally the results; would you get the extension?
Flat tire game

- Why did you choose the tire you did?
- What factors influence choices?
- What factors might allow students to coordinate successfully?
- This is an example of a coordination or “focal point” game
Flat tire game – normal form

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Flat tire game – extensive form

• See drawing on board
• Pure coordination games like this have certain characteristics.
  – Payoffs are dependent on others’ choices
  – There is no conflict of interest
Generic types of games

- Pure coordination
- Zero-sum: the opposite of pure coordination, where interests are in direct conflict, like divide-the-dollar
- Enforcement problems: structure of the game leads to a suboptimal outcome, so enforcement is needed to make everyone better off. Common examples are commons problems.
- Note: in this class, will focus on self-enforcing equilibria
Games we will study

• Most games combine elements of these 3 simple games; will have some conflict of interest, some benefit from coordination, some enforcement problems. For example, consider explicit or tacit bargaining situations.

• Will first study the basics of how to illustrate and solve games

• Then study simple types of games

• Then think about how to put them together
Claim a pile of dimes

• Two players, A and B
• I will put one dime on the table
• Player A can say Stop or Pass
• If Stop, then A gets the dime and the game ends
• If Pass, I put another dime on the table and it’s B’s turn to say Stop or Pass
• Game will end when there is one dollar on the table (players get maximum of 5 turns each)
Claim a pile of dimes

• Results from playing this with 5 different pairs
• Why did players do what they did?
• Did players learn from earlier rounds of the game?
Bargaining game

• Two players, A and B
• A offers a split of a dollar (whole dimes only)
• If B accepts, both get paid and the game ends
• If B rejects, B gets to make an offer, but now the amount to be split is only 80 cents
• If A accepts, both get paid and the game is over.
• If A rejects, game is over and neither get anything
Bargaining game

• Repeat with second-round total falling to 70, 60, 50, and 40 cents
• Tally results
• Why did players behave as they did?
• How did the falling payoffs matter?
• What would the equilibrium be?
Rationality

• Rationality one of the major assumptions of game theory.

• Definition: rationality means
  – Preferences are complete
  – Preferences are consistent
    \[ A>B, B>C \Rightarrow A>C \]
Rational Choice

- Each player chooses the strategy that leads to the outcome they most prefer. (“optimizing behavior” – Shepsle)
What rationality does NOT mean

• Selfish
• Omniscient
• Short-run
• Sharing the same value system as other players or “ethical people”
• No particular content
• No assumption about ranking of outcomes except that they’re consistent
Is rationality a good assumption?

Potential problems:

• What is the unit behaving rationally?
  – Voter
  – Politician
  – Agency
  – Congress

• The rationality assumption may be harder to accept for non-unitary actors (Arrow)
Is rationality a good assumption?

• Individuals may lack the ability to make the complex calculations required by game theory:
  – May be “boundedly rational” (Simon 1957) or use “fast and frugal heuristics” (Gigerenzer & Todd 1999)
  – May make mistakes in calculations
  – May act on the basis of emotion rather than calculating expected values
How can we justify the rationality assumption?

• Experimental evidence

• General idea of being motivated by goals and trying to do as well as possible seems reasonable

• Will be thinking of players as implicitly choosing optimal strategies, even if they do not go through the actual process of calculation
Sets and probabilities

To fully understand expected utilities and work with them, you need to know something about sets and probabilities.
Sets

Sets: will come up in a number of contexts as we begin to study game theory.

Sets of:

• outcomes
• strategies
Sets

- Definition: a set is a collection of elements.
- The set of all elements is called the universal set; \( U \).
- If element \( x \) belongs to set \( S \), \( x \) is a member of \( S \); \( x \in S \).
- The set containing no elements is called the empty or null set; \( \emptyset \).
Sets

- If all members of $S_1$ are also members of $S_2$, we say that $S_1$ is a *subset* of $S_2$ and that $S_2$ *contains* $S_1$; $S_2 \supset S_1$.

- Sets are *disjoint* if they have no members in common.
Sets

- There are 3 basic operations in set theory:
  - The *union* of $S_1$ and $S_2$, $S_1 \cup S_2$, is the set of all elements that are members of both.
  - The *intersection* is all elements that are members of both; $S_1 \cap S_2$.
  - The *complement* is the set of all elements that are not members of $S_1$; $S_1^c$.
Examples of manipulating sets

- $S_1 = \{a, b, c\}$, $S_2 = \{d, e, f\}$, $S_3 = \{c\}$, $\Omega = \{S_1, S_2, S_3\}$
- $S_1$ and $S_2$ are disjoint
- $S_1 \cup S_2 = \{a, b, c, d, e, f\}$
- $S_1 \cup S_3 = \{a, b, c\}$
- $S_1 \cap S_2 = \emptyset$
- $S_1 \cap S_3 = \{c\}$
- $S_1^c = \{d, e, f\}$
Probabilities

• A probability is a number between (and including) 0 and 1.

• Consider the set of events X in which you are interested; these might be all the possible outcomes of a game.

• Divide this set into some number of subsets Y, Z, …, none of which overlap; which are disjoint.
Probabilities

- The probabilities of each subset occurring must sum to the probability of the full set of events. If that set of events includes all the possible outcomes, its probability is 1.
- Put another way: if the occurrence of X requires the occurrence of *any one* of several disjoint Y, Z, …, then the probability of X is the sum of the separate probabilities of Y, Z, …
Probabilities

- Addition rule: \( p(X) = p(Y) + p(Z) + \ldots \), if \( Y, Z, \ldots \) are disjoint (mutually exclusive) and exhaustive.
- \( p(\emptyset) = 0 \)
- Conditional probabilities: the probability of \( Z \) given \( Y \) is written \( p(Z|Y) \).
Probability example

- Consider a game where the possible outcomes are win, lose, or tie (W, L, T)
- \( P(\text{no } T) = p(W) + P(L) \)
- So, if \( P(W)=.5, \ P(L)=.4, \) then \( P(\text{no } T)=.9 \)
Expected value

• The expectation of $X$, $E(X)$, is the sum of the possible values of $X$ multiplied by the probability that each occurs
• D&S denote the value of the outcome by $X$
• The payoff can take $n$ possible values, $X_1, X_2, \ldots, X_n$.
• The respective probabilities are $p_1, p_2, \ldots, p_n$.
• $EU=p_1X_1+p_2X_2+\ldots+p_nX_n$
  $=\Sigma x_ip(x_i)$, for $i=1$ to $n$. 
Expected utility

• The expected utility of any particular choice is calculated by considering
  – the probability of each outcome associated with the choice
  – the utility attached to each outcome

• Careful! D&S use the term “expected value” interchangeably with “expected utility”; could also use the term expected payoffs.
  – Problem if utility ≠ value or payoff
Decision Theory Examples

• Which is better?
  – Heads: $6, Tails: $1
  – Heads: $5, Tails: $3
  – 6 sided dice: $1 for each point
  – 5 apples
  – 2 apples, 2 bananas
Decision Theory Examples

• Which is better?
  – $100 tax credit for all Students with loans
  – 2 fewer deaths in Iraq

• Which is better?
  – 10 minutes faster in airport security line
  – 1 less plane crash
Decision Theory Examples

- Should Hillary Clinton oppose the war in Iraq?
- Suppose Hillary Clinton wishes to maximize her votes
Decision Theory Examples

1. Identify the options available (support, oppose)
2. Identify the possible outcomes (war goes badly, war goes well)
3. Identify probability of each outcome \((p_W, p_B)\)
4. Identify the utility of each option \((u_{W,S}, u_{B,S}, u_{W,O}, u_{B,O})\)
5. Calculate the expected utility of each option (general formula is)
   \[ EU_S = p_W u_{W,S} + p_B u_{B,S} \]
   \[ EU_O = p_W u_{W,O} + p_B u_{B,O} \]
Comparing expected utilities

- To identify the best choice, find the expected utility of each option and choose the one with the highest expected utility.
- Suppose $u_{W,S} > u_{W,O}$ and $u_{B,O} > u_{B,S}$
- Under what conditions is it rational to oppose the war?
Factors that go into making this kind of decision

- Probability of success or failure
- Expected costs of different policies
- Expected benefits of success
- Comparison of alternatives along these dimensions
Advantages of formalization

• Make elements of the situation explicit:
  – Payoffs
  – Assumptions about risks and probabilities
  – Options available to players
  – Who the players are
  – What information the players have
  – Constraints

• Disaggregate the effects of different policy choices for more precise analysis
Uses of decision theory and game theory

• Predictive
• Normative (prescriptive)
• Explanatory (why)
Decisions versus games

• In both decisions and games, assume rational, optimizing behavior

• In *decision theory*, don’t have to take into account the likely reactions of others. Often true when dealing with a large number of actors, as in a market.
Decisions versus games

• Games, or strategic games, occur when it is necessary to take into account the responses of others.
• Everyone is aware that others are taking their responses into account.
• These are called strategic interactions; game theory is the principal tool for studying them.
Strategic interaction

- What do we mean by strategic interaction?
  - Interactive decisionmaking
  - Actors are aware of their objectives or preferences
  - Actors are aware of limitation on actions; constraints
  - Actors are rational and optimizing
  - Actors realize that they are interacting with other rational and optimizing decisionmakers
Examples of strategic interaction

- Electoral politics
- Global warming
- Iran-Iraq War
- Cuban Missile Crisis
- Nuclear Strategy
Principles of strategic interaction

- Look to the end of a game and reason backward. This is called backwards induction or rollback.
- Focal points: commonly expected strategy on which actors can coordinate
Assumptions

In order to use game theory to study politics, we need to make some assumptions:

• Relations between political actors are driven by strategic considerations. These can be both domestic and international.

• Make assumptions about who the actors (units of analysis) are: voters, leaders, states, NGOs, private market actors, domestic groups, domestic institutions, international institutions.
Timing of moves

• A move is the action that a player can take at some point in the game.
• Moves can be *sequential*: one player goes first, the other responds.
• Sometimes moves are essentially *simultaneous*.
• Also characterize moves as simultaneous if the second player doesn’t know what the first has done.
Conflict of interest

• Players’ interests may be in total conflict: zero-sum or constant-sum game. This occurs if some quantity needs to be divided up; a gain for one is a loss for another. A purely distributional problem.

• Usually in economics and politics, deals are possible that make all players better off; game is positive-sum.

• Games that combine conflict of interest and benefits from cooperation are called mixed-motive.
Repetition

- Some games are one-shot; played only once.
- Some games played repetitively.
- Most political interactions unfold over time.
- Games played over time create new dynamics: reputation; trading off short-term benefits and long-term costs.
Information

• Do both players have full information about the probabilities and payoffs in the game? If so, this is called a full or complete information game.

• If not, this is a game of incomplete information.
Information

If information is incomplete, do both players share the same estimates of probabilities and payoffs?

• If yes, this is a game of symmetric information.
• If no, this is a game of asymmetric or private information.
Information

• In asymmetric information games, one player knows something the others don’t.
• Common case: I know my own payoffs, but you don’t.
• Bluffing, signaling, and screening become central in situations of asymmetric information.
Information

• Signaling occurs when a better-informed player takes steps to reveal information.

• Screening is used by the less-informed player.
Enforceability

- Sometimes external (third-party) enforcement is available, e.g. courts to enforce contracts.
- This can increase the range of mutually-beneficial deals that can be reached; the “right to be sued.”
Enforceability

• In politics, third-party (neutral) enforcement is frequently not available.

• So any enforcement has to be done by the players themselves; agreements must be self-enforcing.

• Games with external enforcement are called *cooperative*, those without are *non-cooperative*. But this doesn’t mean cooperation can’t emerge. We will focus on non-cooperative games.
Strategy

A strategy is a complete plan for playing the game.

• If game is one-shot with just two possible actions, the description of the strategy is simple.

• If repetition or multiple actions, complex because you have to specify how to act in every conceivable contingency.

• Rule of thumb: you have appropriately described a strategy if you could write it down for someone else, and they would play exactly the same way as you.
Payoffs

A payoff is the value associated with each possible outcome.

• Payoffs tell us whether the game is zero-sum, about coordination, or mixed-motive.

• Sometimes, adequate just to rank outcomes (1, 2, 3). At other times, need more precision and so have to attach a numerical scale to outcomes (like dollars).
Payoffs

• Payoffs have to capture everything associated with an outcome that an actor cares about.

• In assigning payoffs, sometimes have to think about players’ attitudes toward risk. Are they risk-averse or risk-neutral?
Common knowledge of rules

Players must all share a *common understanding of the rules of the game*, even if there is incomplete information in the sense discussed earlier. Need to agree on:

- what moves available
- order of moves
- who is playing
- that all players are rational
- Also, know that others share all these beliefs, and they know that you do, etc.
Common knowledge

• Definition (more formally): Any information that all players know, that all players know that all players know, and so on, is *common knowledge*.

• In a game, all of the following is common knowledge:
  – What moves are available
  – Who moves at each point
  – If chance is involved, what the probabilities are
  – What uncertainty all players have
  – What the outcomes are
  – That all players are rational
Equilibrium

The point of game theory is to identify equilibria.

• Definition: each player is using the strategy that is a *best response* to the strategy being used by the other player.

• I.e., given the strategy of each, neither has an incentive to *unilaterally* change their strategy.
Equilibrium

• The existence of an equilibrium doesn’t mean that nothing changes; moves may shift over time, for example, in response to the revelation of new information.

• Everyone may not be happy in equilibrium; it might even be possible for everyone jointly to do better.

• Sometimes determining which strategies are equilibria is difficult, even if concept is simple.
Writing down a game

- Consider a trade game: Congress and the president are deciding whether trade policy should be protectionist or free trade
- President is for free trade
- Congress is more protectionist
- Both prefer a moderate level of protection to the highly protectionist status quo
- Partisan conflict: the president benefits from getting Congress to vote against a moderate bill
Trade game

Order of moves:

• President presents a bill to Congress. The bill can be either free trade or moderate.
• Congress then votes “yes” or “no”
• If no, the protectionist status quo prevails
• See representation on board
Trade game

- This is a sequential move game; the order of moves is clearly defined

We can show this game with a game tree:

- Trees are the *extensive form* of a game
- Trees are made up of *nodes* and *branches*
Game trees

• Two types of nodes
  – Decision nodes
    • Decision nodes are associated with the player who makes a decision there
    • The first decision node is called the initial node, where the game starts
  – Terminal nodes
    • These indicate the possible outcomes and the payoffs attached to them. May be a lot.
Game trees

• Branches represent possible actions from any decision node
• The branches must account for all possible actions at that node (perhaps including doing nothing)
• There must be at least one branch from each decision node; there is no maximum number
• Each decision node can have only one branch leading to it
Strategies

• Definition: a strategy is a complete plan for playing a game

• A pure strategy is a rule that says what choice to make at each decision node; what to do under every contingency, no matter how unlikely
Strategies

- In this game, the president has two pure strategies: free or moderate
- For Congress, the strategy set is more complex, since it is contingent on what the president has done
Strategies

• Congress has 4 pure strategies
  – Yes, if P free or moderate
  – Yes if P free, No if P moderate
  – No if P free, Yes if P moderate
  – No if P free or moderate

• Represent these 4 strategies as Y if F, Y if M; Y if F, N if M; N if F, Y if M; N if F, N if M.
Find the equilibrium

• Definition of equilibrium: all players’ actions are the best response to the actions of others.

• To find an equilibrium, have to look ahead and reason back.

• This is called rollback or backward induction.

• Start by thinking about what will happen at last decision nodes, roll back to initial node
Find the equilibrium

- On a game tree, identify the best choice at each decision node by drawing an arrow.
- *Prune* branches that will not be chosen.
- This leads to a *rollback equilibrium*.
- Note that the first player’s choice will be conditional on what he expects the second player to do.
Describing equilibria

- Describe an equilibrium by the set of strategies that lead to it.
- Here, Congress’ best strategy is N if F, Y if M; P’s best strategy is M.
- This leads to the outcome at the 3d terminal node, with payoff 3, 4.
Finding equilibria

- Note that in equilibrium, many branches and nodes are missed. But we still have to consider what would happen at these in order to identify the equilibrium.
- *Out-of-equilibrium* expectations are important.
- Note that rollback works because players know what all the options are at all nodes (common knowledge).
Adding complexity

• Add another node to this game, allowing Congress to decide whether to give the president fast-track authority.
• Since Congress now moves first, write down its payoff first.
• Assume that Congress prefers not to delegate, all else equal.
• Could also add a second country
Order of moves

Does it matter who gets to move first? Often, yes.

• Some games have a first-mover advantage.
  – Example of entry-deterrence game
• Other games have a second-mover advantage.
  – Example: cake-cutting.
Adding complexity

• When adding more players or moves, the game tree and description of the full strategy set get complicated very quickly.

• When too complex, have to use techniques to simplify, such as calculating an intermediate value function (estimate the value of continuing down different paths).
Subgames

- At any node of a game tree, think of the part that begins there as a game of its own.
- This is called a subgame.
Subgames

• An equilibrium path is composed of just a few branches and nodes from the full game.
• Other paths through the game are *off-equilibrium* paths.
• Subgames along these paths are called *off-equilibrium subgames*.
Subgames

• The equilibrium path of play is determined by expectations of what would happen if players chose an action off the equilibrium path, moving to an off-equilibrium subgame.

• Rollback requires that players make their best possible choice at every point, including off-equilibrium subgames.
Subgames

• Strategies are complete plans for playing a game, and so have to specify actions off the equilibrium path.

• At any node, only the remaining game is relevant for making choices; only the part of the strategy pertaining to that subgame matters.
Subgame

• This part of the strategy is called the *continuation* of the strategy for the subgame.

• Every continuation must be optimal, even for off-equilibrium subgames.
Subgame perfection

• So, rollback equilibria are also *subgame-perfect equilibria* (SPE).

• Definition: An SPE is a set of strategies, for all players, such that at every node of the game tree, even off the equilibrium path, the continuation of the same strategy in the subgame starting at that node is optimal for the player making the decision there.
Subgame perfection

• SPE requires players to use strategies that constitute a rollback equilibrium in every subgame of the larger game.

• This means that SPE can only rely on promises and threats that are credible; that are an equilibrium in all subgames.
Threats, Promises, and Credibility

• The concept of subgame perfection is central to understanding the role of credibility in politics
• Credibility central to understanding the use of threats and promises
• Threats and promises are mirror images of one another; the same credibility issues arise.
Trust and Legoland

- Trust Game
- Legoland Game
Credible promises

• Promises can be modeled in much the same way as threats

• Consider a game where the US promises North Korea increased aid if it dismantles its nuclear program.
Credible promises

• The same credibility problem arises as in threat games, in that moves are sequential.
• North Korea has to move first, decide whether to dismantle
• Then US has to come through with the promised increase in aid
Credible promises

• What are reasonable orderings of the payoffs?
• US would prefer to get NK’s cooperation, then to renege on the promise of aid
• Knowing this, NK may refuse to dismantle unless US can lock itself in to providing the aid
Credible promises

- Credibility problem is exacerbated by the fact that Congress probably has to sign off on the provision of aid
- How can US increase its credibility; i.e., decrease the payoff for reneging?
  - Consider reputation; not really a one-shot game
  - Lock itself in, e.g. eliminate don’t pay option
Other questions

• What factors should increase the likelihood of successful deterrence? Think about how they might change the payoffs for each outcome.

• What is missing in this simple discussion of deterrence?
What’s missing

• More players
• Repeated play
• More options
• Uncertainty
  – About type, resolute or irresolute
  – If this exists, reputation and signaling become important
Strategic form

• Strategic form games are used to study situations with simultaneous moves.
• This means that players must move without knowing what the other player has chosen.
  – Moving at exactly the same time
  – Not able to observe the other’s move
Strategic form

• So, these are games of *imperfect information* (or *imperfect knowledge*).

• This means that strategies cannot be contingent on the other’s move.

• But still need to consider the game from the other’s perspective, figure out how she is likely to play and your best response.
Showing the strategic form

- Illustrate the strategic form with a *game matrix* (game table, or payoff table).
- Also called the *normal form* of a game.
- The dimensions of the table equal the number of players; so a two-player game is two-dimensional. Will focus on these.
Showing the strategic form

• The row headings are the strategies available to player 1; column headings for player 2.
• The cell lists payoffs for that outcome, with the payoff for the row player first, column player second.
Showing the strategic form

• Can use the strategic form for either zero-sum (constant-sum) or non-zero-sum (variable-sum) games.
  – In 0-sum games, the total payoffs for each cell are fixed. So, as a shorthand, can show only payoffs to the first player.
  – For variable-sum, have to show both payoffs.

• How would we write rock paper scissors?
# Rock paper scissors

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Rock paper scissors
zero-sum representation

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Strategies

• In these simple normal-form games, a strategy is just a choice of a move (row or column).

• This is a complete plan for playing the game
  – You can’t make your move contingent on what the other is doing
  – Only playing one time (one-shot game)
Equilibria

• How do we find an equilibrium? Can’t use rollback.

• But the same idea as using rollback: look for an equilibrium in which each player’s action is a best response to the actions of others.

• This is a Nash equilibrium.
Definition of Nash equilibrium

A Nash equilibrium is a configuration of strategies, one for each player, such that each player’s strategy is best for her, given that all other players are playing their equilibrium strategies.

• That is, no player wants to make a unilateral change in strategy.

• This is the fundamental equilibrium concept for non-cooperative games.
Pure and mixed strategies

• Strategies can be *pure* or *mixed*.
  – Pure strategies are non-random courses of action; each move is specified with certainty. That is, all but one move have a probability of zero; the move chosen has a probability one of being chosen.
  – Mixed strategies specify that a move will be chosen randomly from the underlying set of pure strategies with a specified probability.
Finding Nash equilibria

• How do we find Nash equilibria in strategic-form games? A number of methods; will begin with the simplest but least general.
Dominant strategies

• Definition: A dominant strategy outperforms all other strategies, no matter what choice others are making.

• A dominant strategy doesn’t guarantee you a higher payoff than your opponent, just higher payoffs than you would get using any other strategy.
Dominant strategies

More formally:

• Assume 3 strategies \{ABC, abc\}

• C is dominant iff \( P(C, a) \geq P(A, a) \) and \( P(C, a) \geq P(B, a) \) and \( P(C, b) \geq P(A, b) \) … for all possible pairings of strategies, and with at least one strict inequality.
Dominant strategies

• If all inequalities are strict, we say that C strictly dominates A and B; C is a strictly dominant (strongly dominant) strategy.
• If there are some weak inequalities, C is weakly dominant.
• A and B are dominated strategies; A and B are dominated by C.
Dominant strategies

• If there are only two strategies, identifying which (if any) are dominant or dominated is easy.

• If more than two strategies, becomes more complex; it is possible for one strategy to be dominated, but for none to be dominant.
Dominant strategies

• If you have a dominant strategy, you should always play it.
• So finding an equilibrium is easy; just look for the other’s best response to the dominant strategy.
Elimination of dominated strategies

• If no strategy is dominant, look to see if there are any dominated strategies, and eliminate these.
• This can lead to a unique equilibrium; successive elimination of dominated strategies.
Prisoners’ Dilemma

• In the PD, both players have dominant strategies

• Three essential features to the PD:
  – Each player has two strategies, cooperate or defect
  – Each has a dominant strategy to defect
  – The dominance solution equilibrium is worse for both than the non-equilibrium outcome where both cooperate
Prisoners’ Dilemma

<table>
<thead>
<tr>
<th></th>
<th>Cooperate</th>
<th>Defect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cooperate</td>
<td>3, 3</td>
<td>1, 4</td>
</tr>
<tr>
<td>Defect</td>
<td>4, 1</td>
<td>2, 2</td>
</tr>
</tbody>
</table>
Successive elimination of dominated strategies

• Also called *iterated* elimination.

• How this works: if a strategy is dominated, eliminate it.

• Then analyze the remaining game; are there additional dominated strategies you can remove?

• Repeat this process.
Successive elimination of dominated strategies

- If this process leads to a unique equilibrium, we say that the game is dominance solvable.
- Pizza game example: there is a guaranteed customer base of 3000 for each of two pizza places.
- An additional 4000 customers float.
Pizza game

• Charging a high price leads to a profit of $12/pizza
• Medium-price profit = $10
• Low-price profit = $5
• Floating customers go to the lower-price place, split evenly if the same price is charged
## Pizza game

<table>
<thead>
<tr>
<th></th>
<th>High</th>
<th>Medium</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>High</td>
<td>60, 60</td>
<td>36, 70</td>
<td>36, 35</td>
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<tr>
<td>Medium</td>
<td>70, 36</td>
<td>50, 50</td>
<td>30, 35</td>
</tr>
<tr>
<td>Low</td>
<td>35, 36</td>
<td>35, 30</td>
<td>25, 25</td>
</tr>
</tbody>
</table>
Pizza game

• No dominant strategies
• Low price is dominated
• When we remove this option, we are left with a PD; both have dominant strategies
• Medium-medium is the equilibrium
Using dominance to solve

• There are some problems with relying on dominance to solve games:
  – Sometimes doesn’t lead to a unique equilibrium
  – If some strategies are only weakly dominated, then the order in which strategies are eliminated can change the equilibrium found
  – May identify some, but not all, Nash equilibria
Dominance solution misses an equilibrium

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<tr>
<th></th>
<th>A</th>
<th>B</th>
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<tbody>
<tr>
<td>A</td>
<td>1, 0</td>
<td>0, 1</td>
</tr>
<tr>
<td>B</td>
<td>1, 1</td>
<td>1, 1</td>
</tr>
</tbody>
</table>
Minimax strategies

• The minimax method of finding a Nash equilibrium applies only to zero-sum games.
• It relies on recognizing that whatever is good for one player is bad for the other.
• So, A will anticipate that B will choose the strategy that is worst for A.
• Knowing this, A should choose a strategy that provides the best of these bad outcomes.
• That is, A maximizes the minimum payoff.
Cell-by-cell inspection

- Cell-by-cell inspection always works to find all Nash equilibria in pure strategies.
- But it can be tedious.
- Sometimes it is the only method that works.
- For each cell, ask whether either player has a unilateral incentive to move.
Cell by cell inspection

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>C</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>1,4</td>
<td>5,8</td>
<td>4,3</td>
</tr>
<tr>
<td>M</td>
<td>3,2</td>
<td>2,2</td>
<td>6,1</td>
</tr>
<tr>
<td>D</td>
<td>7,0</td>
<td>3,6</td>
<td>1,9</td>
</tr>
</tbody>
</table>
Multiple equilibria

• Games with multiple equilibria are usually known as *coordination* games.
• Using only the concept of Nash equilibrium, there is no way to know which outcome will prevail.
• However, we can eliminate all non-equilibrium outcomes.
Coordination games

• Many games in politics are about coordination like the flat tire game
• Coordination games may be less difficult to solve in practice than in theory.
• May need to turn to theories of cognition and perception to understand how they are solved.
Pure coordination game

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<th>A</th>
<th>B</th>
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<tbody>
<tr>
<td>A</td>
<td>1, 1</td>
<td>0, 0</td>
</tr>
<tr>
<td>B</td>
<td>0, 0</td>
<td>1, 1</td>
</tr>
</tbody>
</table>
Coordination problems

• Can think of coordination problems as being differentiated by how much conflict of interest there is in the game.
  – Do all agree on which equilibrium is the best?
  – Or do they disagree?

• Three important examples: Assurance, chicken, battle of the sexes
**Assurance (or stag hunt)**

<table>
<thead>
<tr>
<th></th>
<th>Refrain</th>
<th>Build</th>
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</thead>
<tbody>
<tr>
<td>Refrain</td>
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<td>1, 3</td>
</tr>
<tr>
<td>Build</td>
<td>3, 1</td>
<td>2, 2</td>
</tr>
</tbody>
</table>
Assurance

- In assurance games, players should be able to get to their preferred outcome unless one is uncertain that the other really has assurance preferences.
- Example: arms races
- If preferences are known, all could just unilaterally disarm.
- Might solve by sharing information
## Chicken

<table>
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<th>Swerve</th>
<th>Straight</th>
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<tr>
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</tr>
<tr>
<td>Straight</td>
<td>1, -1</td>
<td>-2, -2</td>
</tr>
</tbody>
</table>
Chicken

• Here, the player who can make a commitment first will “win” – for example, throwing the steering wheel out of the window.

• But if both make commitments, leads to disaster.
**Battle of the sexes (or two cultures)**

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<tbody>
<tr>
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<td>0, 0</td>
</tr>
<tr>
<td>B</td>
<td>0, 0</td>
<td>1, 2</td>
</tr>
</tbody>
</table>
No Nash equilibrium in pure strategies

- Sometimes no matter which outcome is chosen, one player has an incentive to move away from it.
- This means there is no Nash equilibrium in pure strategies.
- The essence of such a game is that there is a benefit to being unpredictable.
No Nash equilibrium in pure strategies

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<td>2, 3</td>
<td>3, 2</td>
</tr>
<tr>
<td>B</td>
<td>4, 1</td>
<td>1, 4</td>
</tr>
</tbody>
</table>
No Nash equilibrium in pure strategies

• Games like this might arise between a defender and an attacker; if the defender knows what the attacker is going to do, can take advantage of it.

• So the solution is to be unpredictable; to choose moves randomly.

• This is called a *mixed strategy*.
# Three player games

Player 3 chooses:

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<thead>
<tr>
<th></th>
<th>Cooperate</th>
<th>Defect</th>
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</thead>
<tbody>
<tr>
<td><strong>C</strong></td>
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<td>3, 6, 3</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>6, 3, 3</td>
<td>4, 4, 1</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Cooperate</th>
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<tr>
<td><strong>C</strong></td>
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<td>1, 4, 4</td>
</tr>
<tr>
<td><strong>D</strong></td>
<td>4, 1, 4</td>
<td>2, 2, 2</td>
</tr>
</tbody>
</table>
Cooperation, coordination, and competition in politics

• Simple 2x2 games have been used extensively in the study of politics.
• Basic problems of politics are: cooperation, coordination, and competition.
• When does each occur?
• What kind of framework allows solution of each?
Strategic form games and politics

- Example of an important argument: the type of framework needed to deal with particular issues depends on the underlying game being played.

- In PD, players have a common interest in cooperation but incentives to defect.

- This is a typical collective-goods problem; also used for trade, arms control.
Strategic form games and politics

• So, in a PD, need a framework to provide monitoring and sanctioning.
• This can be done with institutions that facilitate collaboration
  – The police
  – The state
  – The constitution
  – The UN
Strategic form games and politics

• Coordination problems mean that some outcomes are disliked by all actors.

• But there is a multiple equilibria problem, and possibly disagreement on which equilibrium is preferred.

• Here, an institution would only need to help players identify a focal point; no need for enforcement.
Strategic form games and politics

• Can also use simple games to generate predictions about likelihood for conflict or cooperation.

• What types of games are most likely to give rise to recurrent conflict? To cooperation? How do games map onto different issue-areas?
Group Choice Analysis

• Three friends choose an activity
• Andrew
  – 1) MFA, 2) WP, 3) RS
• Bonnie
  – 1) WP, 2) RS, 3) MFA
• Chuck
  – 1) RS, 2) WP, 3) MFA
• How to decide?
Majority Rule

- All against all – three way tie
- MFA vs. WP
  - WP wins 2-1 (B,C)
- MFA vs. RS
  - RS wins 2-1 (B,C)
- WP vs. RS
  - WP wins 2-1 (A,B)
- Which majority should we use?
Lessons

• Multiple majorities
• Sincere vs. Strategic Choice
  – Misrepresentation of preferences
  – Control of the means of decision
• Multiple decision rules
  – Unanimity
  – First-preference majority rule
  – Majority rule by round robin
Slight Change in Preferences

• Andrew
  – 1) MFA, 2) WP, 3) RS
• Bonnie
  – 1) WP, 2) RS, 3) MFA
• Chuck
  – 1) RS, 2) MFA, 3) WP
• Now what?
Majority Rule

• MFA vs. WP
  – MFA wins 2-1 (A,C)
• MFA vs. RS
  – RS wins 2-1 (B,C)
• WP vs. RS
  – WP wins 2-1 (A,B)
• No winner!
Agenda Procedures

• Round robin fails due to a *cycle* of preferences (*intransitive*)
• What about an agenda procedure?
Choose an Agenda

• Agenda 1
  – MFA vs. WP, then winner vs. RS
• Agenda 2
  – MFA vs. RS, then winner vs. WP
• Agenda 3
  – RS vs. WP, then winner vs. MFA
• Which would Andrew choose?
Rational People, Irrational Groups

• Even when rational individuals honestly reveal their preferences, group's preferences may be intransitive

• This is an instance of what political philosophers Brian Barry and Russell Hardin call “rational man and irrational society”
No Objective, Rational Group Choice

• A group of rational individuals can collectively produce irrational results.

• As a consequence, what is best for the group, or even what a majority thinks is best for the group, is not at all evident.

• Institutional procedures critical in group choice.
Condorcet’s Paradox

- Group of at least three people must choose from at least three alternatives
- One prefers a to b, another prefers b to c, and another prefers c to a.
- What should the group do?
- What does the group do?
Is this really a problem?

• Suppose three alternative
• Six orderings:
  – a to b to c
  – a to c to b
  – b to a to c
  – b to c to a
  – c to a to b
  – c to b to a
Frequency of cycling

- $6 \times 6 \times 6 = 216$ possible “societies” of three individuals

- How many cycles?
  - Three forward cycles (abc, bca, cab)
  - Three backward cycles (acb, cba, bac)

- Only 12 of 216 societies have a cycle

- Good news! Majority rule works in the other 204 societies
Larger groups, more alternatives

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>9</th>
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<td>.200</td>
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<td>1.00</td>
<td>1.00</td>
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</tr>
</tbody>
</table>
Now the Bad News

• We can only use Majority Rule if we are willing to tolerate
  – Small number of voters
  – Restricted agenda access
  – “Irrational” group decisions

• Correlation of preferences mitigates problem somewhat, but not in large groups with many alternatives
Distributional Games

- Often in politics we must divide a fixed amount like a budget
- Suppose politicians from East, West, and North Davis must divide $3000 in spending
- Is there a Condorcet cycle in this game?
Condorcet and Davis Politics

- Suppose three alternatives [N,E,W]:
  1) Three way fair share [1000,1000,1000]
  2) Two way fair share [1500,1500,0]
  3) Another possibility [2500,0,500]
- 2 beats 1 (N,E)
- 3 beats 2 (N,W)
- 1 beats 3 (E,W)
Implications

• ALL DISTRIBUTIONAL GAMES CYCLE!
• Only solution is an anti-majoritarian restriction
• This is the core of Arrow’s theorem:
  – Condorcet paradox extends to any “reasonable” method of aggregating group preferences when there are at least 3 rational individuals and at least 3 alternatives
Arrow’s Theorem

- Four conditions:
  1) **Universal admissibility**
     (any individual preferences are allowed)
  2) **Pareto Optimality**
     (unanimous preference $\rightarrow$ group preference)
  3) **Independence from Irrelevant Alternatives**
     (adding alt’s does not change group pref.)
  4) **NonDictatorship**
     (no individual’s pref. determines group pref.)
Arrow’s Theorem

• There exists no mechanism for translating the preferences of individuals into a coherent group preference that satisfies U, P, I, and D

• In other words, any mechanism that satisfies U, P, and I is either dictatorial or incoherent!

• Tradeoff between diffusion of power and coherence
Applying Arrow to Majority Rule

• MMR: Group prefers $j$ to $k$ iff a majority of individuals prefer $j$ to $k$

• Three reasonable conditions:
  – **Anonymity**
    (no dependence on identity of individuals)
  – **Neutrality**
    (no dependence on identity of alternatives)
  – **Monotonicity**
    (individual change must yield consistent group change)
May’s Theorem

• A method of preference aggregation is MMR iff it satisfies conditions $U, A, N, M$.

• If you think majority rule is “fair” then you must also think each of these conditions is “fair”
May and Arrow

• May is a special case of Arrow!
  – Unanimity in both
  – Anonymity $\Rightarrow$ Nondictatorship
  – Neutrality $\Rightarrow$ Independence from Irrelevant Alt.
  – Monotonicity $\Rightarrow$ Pareto Optimality

• Thus, Arrow applies to MMR

• Which should we relax to understand group choice?
Single-Peakedness

• Suppose we relax Universality
• Then we can get Black’s Theorem: *If, for every subset of three alternatives, one of these alternatives is never least preferred by any group member, then MMR yields transitive group preferences.*

• In other words, we restrict preferences to be single-peaked
Value Restrictions

- Sen generalizes this result, noting that MMR yields coherent group preferences if there is agreement about any rank restriction
- But this essentially assumes away cycles
- If cycles exist, value restrictions fail
Cuban missile crisis

• Three options for the U.S. response to Soviet missiles:
  1) Diplomacy
  2) Blockade
  3) Invasion

• Three groups of advisors:
  1) Hawks: I>D>B (JCS, Acheson)
  2) Doves: D>B>I (Stevenson)
  3) Statesmen: B>I>D (RFK, McNamara)
One-dimensional spatial model

• In order to show a group decision problem spatially, need to specify how policy options are arrayed on one dimension
  – The *ideal point* (the most preferred outcome) for each individual
  – The utility for each outcome is represented by curves moving away from the ideal point
Cuban missile crisis

Utility

Doves

Statesmen

Hawks

Aggressiveness

D

B

I
Single-peaked preferences

• The “problem” with the Cuban missile crisis example is that the Hawks’ preferences are not single-peaked.

• But many choices are, so we will frequently assume
  – each individual has a single most preferred outcome
  – the utility from other outcomes decreases with the “distance” from the most preferred outcome
Let’s Play Some Games

• 2 office-motivated candidates, 1 issue
• 2 office-motivated candidates, 2 issues
• 2 policy-motivated candidates, 1 issue
• 3 office-motivated candidates, 1 issue
• 3 office-motivated candidates, 2 issues
• 1 policy-motivated agenda setter, multiple votes, 2 issues
Theory of Committees and Elections (Black 1958)

- Committee chooses one of several alternatives
- Each committee member has a unique preferred alternative and single-peaked preferences
Theory of Committees and Elections (Black 1958)

- Proposition
  - Optimum of median member defeats any other alternative in pairwise voting under majority rule

- Prediction
  - Median committee member’s preference will be the decision of the committee

- What does it mean if this prediction is falsified?
Median Voter Theorem (Downs 1957)

- Prior to Black, Downs imports a model from economics by Hotelling (1929)
- Two stores decide where to locate on a street
- Suppose shoppers are lazy and go to the store closest to them—where will the stores locate?
- Equivalently, if voters choose the closest of two policies, where will parties/candidates locate?
Median voter theorem

- Note that the median voter is not usually the same as the mean voter.

\[\text{Median voter theorem}\]

\[\text{Mean}\quad\text{Median}\quad\text{Income}\]
Median voter example

• C is the median voter below
• A proposal at C will beat any other proposal, even $x_3$

<table>
<thead>
<tr>
<th></th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>$x_3$</th>
<th>$x_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
</tr>
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</table>
Multiple dimensions

- Linked issues lead to multidimensional decisions; issues often are not decided one at a time.
- For example, if deciding environmental policy, economic growth may be a concern.
- Or military and economic policies could be linked.
Multiple dimensions

- To illustrate two linked issues, show two dimensions (x and y, horizontal and vertical)
- Show each actor’s ideal point in this two-dimensional space.
- Consider *indifference curves*: curves that indicate sets of points among which the actor is indifferent.
Indifference curves

- If the actor gives the same weight to both dimensions, indifference curves will be circular.
- The space contains an infinite number of indifference curves.
- The indifference curves that run through the status quo are especially interesting, because they show us the set of points that the actor prefers to the status quo.
Sample Exam Questions

• Use strategic form to represent a game of chicken between two players. What are the equilibrium strategies and outcomes? Briefly describe how this game might help us understand a problem in political science.

• What is Arrow’s Impossibility Theorem? What are its implications for political competition? for normative Democratic theories?
Indifference curves

Butter

Guns

A

Q
Indifference curves

• The circle that runs through Q and has an actor’s ideal point at the center shows the points that the actor prefers to Q – the interior of the circle.

• This is the actor’s preferred to set.
Win sets

• If moving policy away from the status quo requires the approval of two actors, the new policy must be in the overlap of their preferred-to sets.

• This overlap is called the *win set*: the set of points that can beat the status quo.
End of Cold War example

- If agreeing on a policy requires 2 out of the 3 factions, 3 win-sets exist.
- The G-Y win-set is substantially larger than G-L or L-Y. So there are more points that G and Y could agree on.
- Policies within the G-Y win-set will involve more dovish foreign policy, but not necessarily more market orientation.
- Multiple votes = no equilibrium….
When is there equilibrium?

• Plott’s Theorem:
  – If an odd number of voters possess distance-based spatial preferences and if their ideal points are distributed in radial symmetry to a given alternative, then it cannot be defeated by any other alternative in a pairwise vote.

• Problem: extremely rare
McKelvey’s Chaos Theorem

• *If there is more than one issue and radial symmetry does not exist then there is no alternative that will defeat all other alternatives. Anything can happen and whoever controls the order of voting can determine the final outcome*
Legislative Outcomes

• Let’s go back to the one dimensional case
• What is the outcome of any vote in a legislature that uses majority rule?
• Suppose there are legislative committees
  – and there is an *open rule* (amendments can be made in the legislature)
  – and there is an *closed rule* (amendments can *not* be made in the legislature)
Legislative Example

Legislative winset

Committee Winset

$x^0$  $x^c$  $x^*$  $x^m$
Legislative outcomes

• Majority rule
  – Median legislator’s ideal point

• Open rule
  – If median legislator is closer to median committee member than to status quo, then median legislator’s ideal point
  – Otherwise status quo
Legislative outcomes

- **Closed rule**
  - If median committee member is between status quo and median legislator, then *median committee’s ideal point*
  - If status quo is between median committee member and median legislator, then *status quo*
  - If median legislator is between status quo and median committee member, then it is more complicated
Two Legislative Examples

Legislative winset

Committee Winset

Legislative winset

Committee Winset

$x^0 \quad x^m \quad x^* \quad x^c$

$x^0 \quad x^m \quad x^c$
# Hypothetical Electorate

<table>
<thead>
<tr>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
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</table>
Voting Methods

- **Simple plurality**
  - Each voter casts one vote, highest total wins

- **Plurality runoff**
  - Two alternatives with highest total compete in another round of voting

- **Sequential runoff**
  - Subset of alternatives with highest total compete in another round of voting
Voting Methods

• Borda count
  – $N$ alternatives, each voter ranks alternatives, $N$ points awarded to first alternative, $N-1$ to second, and so on

• Condorcet procedure
  – Pairwise round-robin tournament, alternative winning majority of contests wins

• Approval voting
  – Each voter approves of 1 to $N$ alternatives, alternative with most approval votes wins
Voting Methods

• Each of these methods produces a different winner in the hypothetical electorate!

• What demands does each make on voter decision-making?

• What demands does each make on voter time?

• Does each method always yield a winner?

• Which of these is “fair”? Which most accurately represents the “general will”?
Electoral Systems

- Plurality voting
  - Citizens vote for individuals and top vote-getters win seats

- Proportional representation
  - Citizens vote for parties and seats are allocated in proportion to the vote polled by each party
Electoral Systems

• Five Pieces of Information
  – Number of votes per voter ($v$)
  – May voters partially abstain? ($p$)
  – May voters cumulate their votes? ($c$)
  – Number of legislators per district ($k$)
  – Electoral formula ($f$)
## Some Plurality Systems

<table>
<thead>
<tr>
<th>System</th>
<th>$v$</th>
<th>$p$</th>
<th>$c$</th>
<th>$k$</th>
<th>$f$</th>
<th>where?</th>
</tr>
</thead>
<tbody>
<tr>
<td>First Past the Post (FPP)</td>
<td>1</td>
<td>no</td>
<td>no</td>
<td>1</td>
<td>Plurality</td>
<td>US, UK</td>
</tr>
<tr>
<td>Single Nontransferable Vote (SNTV)</td>
<td>1</td>
<td>no</td>
<td>no</td>
<td>$&gt;1$</td>
<td>Plurality</td>
<td>Past Japan</td>
</tr>
<tr>
<td>Limited Vote (LV)</td>
<td>$&lt;k$</td>
<td>yes</td>
<td>no</td>
<td>$k$</td>
<td>Plurality</td>
<td>Past UK</td>
</tr>
<tr>
<td>Cumulative Vote (CV)</td>
<td>$\leq k$</td>
<td>yes</td>
<td>yes</td>
<td>$k&gt;1$</td>
<td>Plurality</td>
<td>Past Illinois</td>
</tr>
<tr>
<td>Single Transferable Vote (STV)</td>
<td>$\leq k$</td>
<td>yes</td>
<td>no</td>
<td>$k&gt;1$</td>
<td>Plurality</td>
<td>Ireland, Australia</td>
</tr>
</tbody>
</table>
Equilibrium in Plurality Systems

- **Centripetal tendency**
  - Candidates move towards median voter
  - Occurs when there is no cumulation and few candidates per vote
- **Centrifugal tendency**
  - Candidates move away from median voter
  - Occurs when there is no cumulation and many candidates per vote or when there is cumulation
- Empirically verified (Cox)
Proportional Representation

- Most common form of government
- Much variation:
  - How to divide a fixed number of seats?
  - Minimum threshold for representation
  - Who takes office? (direct representation vs. party lists)
Duverger’s “Law”

• FPP yields two party systems
  – third parties have little chance of winning 50% of the vote
  – when they do, they displace existing party

• PR yields multi-party systems
  – Third party can survive with significant minority

• Mixed systems becoming more common
Strategic voting

• You might vote for something other than your most preferred outcome, anticipating that this will lead to a better outcome for you in the end.

• This contrasts with sincere or naïve voting.

• For example, if you expect your preferred outcome (Nader) to lose, but the other two options are tied, you might strategically choose one of the other two.
Pay Raise Game

• You are one of three legislators to vote on a pay raise
• You each prefer
  – more pay to less
  – to vote against the pay raise instead of for
  – getting a pay raise and voting for it to getting nothing
• A motion for a roll call vote comes up and you must vote first—how do you vote?
Manipulation and Strategic Behavior

- We might want a *strategy-proof* decision-making procedure (not manipulable, invulnerable to intentional misrepresentation of preferences)

- Gibbard-Satterthwaite Theorem: Every nondictatorial social choice procedure is manipulable
Implications of Strategic Behavior

• Killer amendments and “heresthetic”
• Misrepresentation of support for third parties in US
• However, the median voter theorem holds even with strategic voting, as long as there is always a head-to-head choice between the median and all other proposals.
Strategic and Extensive Form

• How to represent strategic form games in extensive form
  – Chicken

• How do we represent imperfect information?

• Information Set
  – Set of all decision nodes following an unobserved decision in a game tree
Equilibria

- Sometimes there is a difference between Nash and Subgame Perfect Equilibrium
- SPE is a subset of NE
- There are many equilibrium refinements – we will mainly focus on Nash and SPE
Strategic and Extensive Form

- How to represent extensive form games in strategic form
  - Legoland game
- Must show all strategy combinations
Combination Game

• Chicken with commitment
No Pure Strategy Equilibrium

• Some games have no pure-strategy equilibrium. Instead, choices will keep cycling.

• Attack-defend situations, where one player prefers to coordinate and the other to pick different options.

• Matching pennies
No Pure Strategy Equilibrium

- **Tennis** (Walker-Wooders 2002)
  - If service return is predictable (forehand or backhand), will lose.

- **Penalty kicks** (Chiappori, Levitt & Groseclose 2002)
  - If direction is predictable, goalie will stop

- So, you can be exploited if you behave predictably.
Randomization

• To prevent exploitation, have to keep your opponent guessing.

• So, when no pure strategy equilibrium, have to randomize; use a *mixed strategy*. 
Mixed strategies

• **Mixed strategy**: strategy chosen randomly from the set of pure strategies with a specified probability.
  
  • I.e., you use each pure strategy a certain percent of the time.
  
  • Actually need to randomize to get to an equilibrium; can’t just alternate.
  
  • Every simultaneous-move game has a Nash equilibrium in pure or mixed strategies.
Find a mixed-strategy equilibrium

Consider the following game.

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>U</td>
<td>2, 3</td>
<td>3, 2</td>
</tr>
<tr>
<td>D</td>
<td>4, 1</td>
<td>1, 4</td>
</tr>
</tbody>
</table>
Steps in finding equilibrium

• Payoffs used to find mixed strategies must be cardinal, not just a ranking (ordinal).

• Elements of finding a mixed-strategy equilibrium:
  1. Expected values
  2. Best-response curves
  3. Upper envelope
  4. Value of game = payoff in equilibrium
Calculating \( p \) and \( q \)

- 1 plays up with probability \( p \), down with probability \( 1-p \)
- 2 plays left with probability \( q \), right with probability \( 1-q \)
- Equilibrium means
  - your opponent has no incentive to change his strategy
  - she is *indifferent* between her pure strategies
Calculating p and q

• 1 chooses p so that 2 is indifferent between left and right.
• 2 chooses q so that 1 is indifferent between up and down.
• To find p and q, calculate Column’s expected payoff as a function of p, and Row’s expected payoff as a function of q.
Calculating p and q

- 2’s payoff for choosing left = 3p+(1-p)
- 2’s payoff for choosing right = 2p+4(1-p)
- For 1 to make 2 indifferent, set these equal to one another and solve for p:
  - 3p+1-p = 2p+4-4p
  - 2p+1=4-2p
  - 4p=3
  - p=3/4
- If 1 chooses p=3/4, 2 will be indifferent between left and right.
Calculating $p$ and $q$

- 1’s payoff for choosing up = $2q + 3(1-q)$
- 1’s payoff for choosing down = $4q + (1-q)$
- 2 will choose $q$ to make 1 indifferent between these two pure strategies:
  - $2q + 3 - 3q = 4q + 1 - q$
  - $3 - q = 3q + 1$
  - $4q = 2$
  - $q = 1/2$
- Equilibrium: 1 will play up with probability $3/4$, down with probability $1/4$; 2 will play left with probability $1/2$ and right with probability $1/2$. 
Calculating p and q

• What we have done is to calculate each player’s *expected payoff* of each pure strategy as a function of the other player’s probability choice.
• Then set the expected payoff of the pure strategies equal to one another.
• And solve for the other player’s probability choice (p and q).
Best response curves

- Can show the same logic by looking at best-response curves
- Show each player’s payoff to each pure strategy as a function of the other player’s probability choice
- This will produce two lines
- The “upper envelope” created by these two lines shows the best response to the other player’s choice
Best response curves

• Take the two players’ best response curves and plot them on the same graph.

• Where the two lines intersect is the equilibrium; each player’s strategy here is a best response to the other’s.

• This will give the same value of p and q as above.
Mixed strategy equilibrium

- Note that since \( p \) and \( q \) are probabilities, their values must be between and including 0 and 1. If they don’t fall in this range, check your math – or maybe there are only pure strategy equilibria.
Value of the game

• The value of the game to each player is their expected payoff in equilibrium.

• 2’s expected payoff =
  3pq+1(1-p)q+2p(1-q)+4(1-p)(1-q)
  = 3(3/4)(1/2)+1(1/4)(1/2)+
    2(3/4)(1/2)+4(1/4)(1/2)
  = 9/8+1/8+6/8+4/8
  = 5/2

• Value of the game for 2 is 5/2=2.5
Value of the game

- Value of the game for 1 =
  \[2pq + 4(1-p)q + 3p(1-q) + 1(1-p)(1-q)\]
  \[= 2\left(\frac{3}{4}\right)\left(\frac{1}{2}\right) + 4\left(\frac{1}{4}\right)\left(\frac{1}{2}\right) + 3\left(\frac{3}{4}\right)\left(\frac{1}{2}\right) + 1\left(\frac{1}{4}\right)\left(\frac{1}{2}\right)\]
  \[= \frac{6}{8} + \frac{4}{8} + \frac{9}{8} + \frac{1}{8}\]
  \[= \frac{5}{2}\]
## US-Japan trade game

<table>
<thead>
<tr>
<th></th>
<th>US permit imports (q)</th>
<th>US restrict imports (1-q)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Japan builds in Japan (p)</td>
<td>1, 0.5</td>
<td>0, 0</td>
</tr>
<tr>
<td>Japan builds in US (1-p)</td>
<td>0, 0</td>
<td>0.5, 1</td>
</tr>
</tbody>
</table>
Equilibrium in US-Japan game

- Calculate expected payoff to US for Japan’s choice:
  1. If US chooses permit: \( p(0.5) + (1-p)0 = 0.5p \)
  2. If US chooses restrict: \( p(0) + (1-p)1 = 1-p \)

- Japan will choose \( p \) to make US indifferent between these two: \( 0.5p = 1-p \)
  \[ 1.5p = 1 \]
  \[ p = 1/1.5 = 2/3 \]
Equilibrium in US-Japan game

- Calculate expected payoff to J for US choice:
  1. If Japan chooses Japan: \( q(1) + (1-q)0 = q \)
  2. If Japan chooses US: \( q(0) + (1-q)(.5) = .5 - .5q \)

- US will choose \( q \) to make Japan indifferent between these two pure strategies:
  \[ q = .5 - .5q \]
  \[ 1.5q = .5 \quad \Rightarrow \quad q = .5 / 1.5 = 1/3 \]
Best response curves

- Graph payoffs as function of opponent choice
- Upper envelope = best response
- Equilibrium at the intersection of the best response curves.
- Note that there are 3 points of intersection: 0, 0; 1,1; and 2/3, 1/3
- This is because there are also two pure strategy equilibria in the game
US-Japan game

- Best-response curves show that pure strategies are the best response except at the point where these two curves intersect; this is the mixed-strategy equilibrium.
- If coordinated randomization were possible (e.g., alternating), could improve US expected payoff from 1/3 to 2/3.
- Value of the game = 1/3 for US, 2/3 for Japan (on board)
Mixing in practice

• What does mixing mean in practice? Have to have a device that randomizes for you.

• Walker-Wooders
  – Distribution of service returns should be random.
  – Do see equal probabilities of winning to each side, as expected.
  – But players “switch” too often to be truly random.

• Generate a random integer between and including 1 to 10 – what did you choose?
Mixing in practice

• Schelling uses the example of a threat that leaves something to chance = *brinksmanship*.
• When mixing in a coordination game, you run the risk of arriving at an uncoordinated outcome. So the payoff is generally less than from a pure strategy equilibrium.
• But you may have to take this chance of an occasional poor outcome to get the highest expected value (poker!)
Mixed strategies in politics

• Would a political actor ever really play a mixed strategy?
• What does this mean in practice?
• Schelling suggests that they do, even if not consciously, because the process of decision-making leaves something to chance.
• Techniques like tripwires can also leave something to chance.
Coordination and distribution

- Coordination games have multiple equilibria
- With mixed strategies possible, multiple equilibria will often emerge
- Some equilibria leave all players unhappy
- Distributional conflict arises when players disagree over which of the equilibria they prefer (battle of the sexes, chicken)
The bargaining problem

• Bargaining is extremely important in politics, but we haven’t said much directly about it.
• What exactly is bargaining? How do we represent it using game theory?
• Bargaining is a *coordination* problem.
  – Two parties need to agree on the distribution of a good.
  – Many equilibria, but disagreement over which is preferred (distributional conflict).
Bargaining problems

• Failure to reach an agreement leaves all parties worse off. So bargaining involves:
  1. Potential for mutual gains
  2. Conflict over how to divide these gains
• Bargaining is not zero-sum: a surplus exists, compared to a situation where no bargain is reached.
• Best Alternative to a Negotiated Agreement (BATNA) influences the outcome
Types of bargaining models

• Nash bargaining model
  – Based on cooperative game theory
  – Underspecified
• Legislative bargaining
• Alternating offers
Alternating offers model

• In each period, one player has the opportunity to make an offer to the other.
• The other player can accept or make a counter-offer.
• This process continues until an offer has been made and accepted.
Alternating offers model

• With a finite number of periods, can use rollback to find the equilibrium.
• In an infinitely-repeated game, why would this process ever end?
• Have to assume that the surplus becomes less valuable over time; discounting.
  – This could result because the surplus itself is shrinking (some probability it will disappear, e.g.), or because the players are impatient.
Discounting and Present values

• Suppose you have a choice between $x = 95$ today or $y = 100$ tomorrow—which do you choose?

• If we *discount* tomorrow’s payoffs, we must figure out how to compare them to today’s.
Discounting and Present Values

• The Present Value of tomorrow’s payoff is the payoff times the discount factor (\(d\) or \(\delta\), “delta”)
  \[ PV = \delta y, \quad 0 < \delta < 1 \]

• Higher discount factor = more patient

• Suppose tomorrow’s payoffs are only worth 90% of today’s payoffs (\(\delta = 0.9\))
  – Do you choose \(x\) or \(y\) ?
  – Can you generalize the choice?
Discounting and Interest Rates

• Interest rate \((r)\)

• Suppose you choose \(x\). What interest rate would you need in order to have \(y\) tomorrow?

\[ x (1+r) = y \]

\[ 95 (1+r) = 100 \]

\[ r = 5/95 = 5.26\% \]

• Can you generalize the choice?
Discounting and Interest Rates

- You are indifferent between present and future payoff when \( x = \delta y \) and \( x (1+r) = y \)
- Notice these are just two different ways of thinking about the same problem
- Solving for \( \delta \) and \( r \) yields
  \[
  \delta = \frac{1}{1+r}
  \]
Discounting in politics

• What does the rate of return mean in politics?
  – Chance the game will end
  – Interest rate

• What does discounting mean?
  – Many outcomes are time-dependent
Alternating Offer Bargaining

• Suppose two players are bargaining over the division of a dollar.

• A dollar tomorrow is as good as having only 95 cents today ($\delta = 0.95$).

• Assume BATNAs are zero.
Alternating Offer Bargaining

• Player 1 offers a split \((x, 1 - x)\).
• If Player 2 accepts, the game is over. If she rejects, she counter-offers \((1 - y, y)\).
• What is the equilibrium outcome?
Alternating Offer Bargaining

• Player 1 knows that Player 2 will get at least $y$ in the next round, because $y$ is the equilibrium payoff to the player making the offer.

• Player 1 has to offer something today that is at least as good as getting $y$ in the next round.
  \[ 1 - x \geq 0.95 \, y \]

• However, Player 1 also wants to maximize own payoff (minimize Player 2’s payoff)

• Thus, \[ 1 - x = 0.95 \, y \]
Alternating Offer Bargaining

• *Player 2 faces exactly the same problem in the following round!*
• Thus, $1 - y = 0.95 \, x$
• Solve for $x$ and $y$
  
  $1 - y = 0.95 \, x \implies y = 1 - 0.95 \, x$

• Substitute $1 - x = 0.95 \, y$
  
  $1 - x = 0.95(1 - 0.95 \, x)$

  $1 - x = 0.95 - 0.9025 \, x$

  $x = \frac{0.05}{0.0975} = 0.51$
Alternating Offer Bargaining

- So, the equilibrium is
  - Player 1 offers the split (0.51, 0.49)
  - Player 2 accepts in the first round
- The equilibrium is reached immediately even though an unlimited number of counteroffers are allowed
- The outcome is efficient; the surplus does not decay.
- A first-mover advantage results: the player making the first offer gets more (x > 1/2).
Different Discount Factors

• This example assumed that the two players had the same discount factor \( (\delta = 0.95) \).
• What if the two players have different discount rates?
• For Player 1, let a dollar tomorrow may be worth only 0.90 today. Player 2’s discount factor is 0.95.
  – So Player 1 is willing to accept a smaller amount in order to be paid sooner.
• In equilibrium, who gets less?
Different discount factors

• The new equations are:
  \[ 1 - x = 0.95y \quad 1 - y = 0.9x \]

• Solve for \( x \) and \( y \)
  \[ x = 1 - 0.95y \quad y = 1 - 0.9x \]

• Substitute \( x = 1 - 0.95y \)
  \[ x = 1 - 0.95(1 - 0.9x) \]
  \[ x = 0.05 + 0.855x \]
  \[ x = 0.05 / 0.145 = 0.34 \]
Different discount factors

• So, the equilibrium is
  – Player 1 offers the split (0.34, 0.66)
  – Player 2 accepts in the first round

• Even though Player 1 moves first, she gets less because she is less patient!
Bargaining and Institutions

• Arrow tells us that there is no objectively superior bargain
• McKelvey tells us that bargaining over many issues may be chaotic
• How do people who govern respond to these problems?
• Institutions – rules and norms that regulate social interaction
Division of Labor and Regular Procedure

- Separation of agenda from discussion and decisions
  - Calendar in the US Senate
  - Agenda and Minutes of a meeting
- Reduces transaction-costs
- Strategy-enhancing
- Regular Procedure may be amended or suspended
  - Cloture in the US Senate
Specialization of Labor

- Institutions assign different roles to different individuals
- Organizations tend to become more specialized over time
- US Congress evolved from *ad-hoc* committees to standing committees
- Supreme Court has not specialized
- How do individuals in the groups to which you belong specialize?
Jurisdictions

• Activities are assigned to specific individuals / groups in the organization
• Yields incentive to specialize
• May create greater affiliation across groups
  – US President and foreign leaders
  – University professors
Delegation and Monitoring

• The principal-agent problem
• Delegating power may produce better information and better policy
• However, giving jurisdictional power to specialists may be costly
  – Armed Service committee may favor projects that reward their own districts
• Monitoring may also be costly
  – Police patrols, fire alarms, and fire extinguishers
Legislatures

- Legislators must please many people
  - Donors and volunteers
  - Voters
  - Themselves

- Delegates or Trustees?

- Either way, preferences are heterogeneous and diverse

- Legislators must build consensus
Problems in Legislatures

• Majority Cycles
  – Legislators accept binding institutions in order to influence their most preferred issues

• Matching influence and interest
  – Vote trading, “log rolling”

• Information
  – Undersupplied, may be conflicting
Problems in Legislatures

• Compliance
  – Bureaucracy may not implement intended action

• Leadership
  – Leaders have agenda-setting power
  – Determining who will lead may be problematic
  – Seniority vs. other criteria
The Collective Action Problem

• Consider the prisoner’s dilemma
  – Each person has an individual incentive to defect
  – Each would prefer the cooperative outcome

• The problem only gets worse in a game with many actors

• Consider the act of voting
The Paradox of Turnout

• Actors: voters
• Actions:
  – vote for Left candidate
  – vote for Right Candidate
  – Abstain
• Outcomes:
  – Left candidate wins
  – Right candidate wins
  – Tie
The Paradox of Turnout

• Assumption: Rationality
  – Preferences are Complete
  – Preferences are Consistent

• Benefit of voting: B
  – (Aldrich normalizes this to 1)

• Cost of voting: C

• Probability of affecting outcome: P

• Vote if PB>C
The Paradox of Turnout

- $P = 0$ unless total vote is within one vote of a tie!

![Figure 1. Decision Table for Rational Choice Models of Turnout](image)

<table>
<thead>
<tr>
<th>Actions:</th>
<th>(1) Winning by More than One Vote</th>
<th>(2) Winning by Exactly One Vote</th>
<th>(3) Tied</th>
<th>(4) Losing by Exactly One Vote</th>
<th>(5) Losing by More than One Vote</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) Vote for Preferred Candidate</td>
<td>$1 - C^*$</td>
<td>$1 - C$</td>
<td>$1 - C$</td>
<td>$1/2 - C$</td>
<td>$0 - C$</td>
</tr>
<tr>
<td>(2) Vote for Other Candidate</td>
<td>$1 - C$</td>
<td>$1/2 - C$</td>
<td>$0 - C$</td>
<td>$0 - C$</td>
<td>$0 - C$</td>
</tr>
<tr>
<td>(3) Abstain from Voting</td>
<td>1</td>
<td>1</td>
<td>$1/2$</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

*Entry is payoff in utiles to decision maker. It is assumed that $0 < C < 1/2$, where $C$ is costs of voting. A tie is assumed to be broken by the flip of a fair coin. Utiles are normalized, so that the value of the preferred candidate winning is 1; the value of the opponent winning is 0.*
Example: Paradox of Turnout

• As \( N \) gets large, \( P \) gets very small even for close elections
  – Bush v. Gore 2000, even if you knew the difference in Florida would be 1000 votes or less, then \( P \sim 1/2000 \)

• “True” \( P \) in US Presidential elections is estimated to be about 1 in 10 million

• Suggests voting is “irrational” or there are other un-modeled benefits to voting
Paradox of Turnout

• Formal theory predicts many relationships that have been observed
  – Turnout increases as benefit increases
  – Turnout decreases as cost increases
  – Turnout increases with closeness of election

• Challenge: identify the incorrect assumptions and change them to get these results and significant aggregate turnout
Tragedy of the Commons

• Individual incentives work against provision of public goods

• Solutions:
  – Internal values
  – External enforcement
    • Third party
    • Repeat play
  – Political entrepreneurs